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by

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Abstract

Using a constitutive equation and anti-plane shear field introduced by Knowles, we show that the expressions of Irwin and Achenbach for the energy release rate are not generally valid for non-linear elastic materials.

On Irwin's and Achenbach's expressions for the energy release rate

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1. Introduction.

Several expressions for the dynamic energy release rate, $\mathcal{E}$, for a straight edge crack have been presented more or less on the basis of intuitive arguments. In a two-dimensional field, most simple[1] and notable among these are the expressions given by Irwin [1,2] (see also Erdogan [3])

$$\mathcal{E}(t_o) = -\frac{1}{2}\left(\frac{d}{dt}\right)_{t_o} \int_{C_{t_o t}} \underset{\sim}{s}(\underset{\sim}{x},t_o)\cdot\underset{\sim}{u}(\underset{\sim}{x}-\underset{\sim}{\ell}(t),t_o)\,ds_{\underset{\sim}{x}} \qquad (1.1)^2$$

and by Achenbach [4-7]

$$\mathcal{E}(t) = -\int_{C_t} \underset{\sim}{s}(\underset{\sim}{x},t)\cdot\dot{\underset{\sim}{u}}(\underset{\sim}{x},t)\,ds_{\underset{\sim}{x}}, \qquad (1.2)$$

where $\underset{\sim}{s}$ is the surface traction, $\underset{\sim}{u}$ is the displacement, $C_{t_o t}$ is the portion of the crack generated in the time interval $[t_o,t]$, $\underset{\sim}{\ell}(t) = \underset{\sim}{z}(t) - \underset{\sim}{z}(t_o)$ with $\underset{\sim}{z}(t)$ the position of the crack tip at time t, and C_t in (1.2) is some portion of the fracture plane which contains the tip $\underset{\sim}{z}(t)$.[3]

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[1] In the sense that the expression requires a knowledge of stress and displacement (or velocity) only on the fracture plane.

[2] Cf. Gurtin and Yatomi [8] for a proof valid within the dynamic theory.

[3] Here an integral $C_{t_o t}$ or C_t has the obvious meaning in terms of integrals over the "two faces" of $C_{t_o t}$ or C_t.

In [3] and [5] the above expressions were deduced from an overall energy balance in a neighborhood of the crack tip by regarding the problem as a half space with time-dependent boundary conditions. We note that (1.2) has an ill-defined integrand, while (1.1) is given in terms of a well-defined, integrable function.

It was shown by Freund [9], using the flux integral expression for the energy release rate and applying it to the boundary of a rectangular fixed region $\mathcal{R}$ surrounding the tip (Figure 1), that (1.2) is given in the limit as $\delta \to 0$. Freund then computed (1.2) for some known linear elastodynamic solutions using the result[1]

$$\int_{-\infty}^{\infty} \frac{H(v)}{v^{1/2}} \frac{H(-v)}{(-v)^{1/2}} \, dv = \frac{\pi}{2} , \tag{1.3}$$

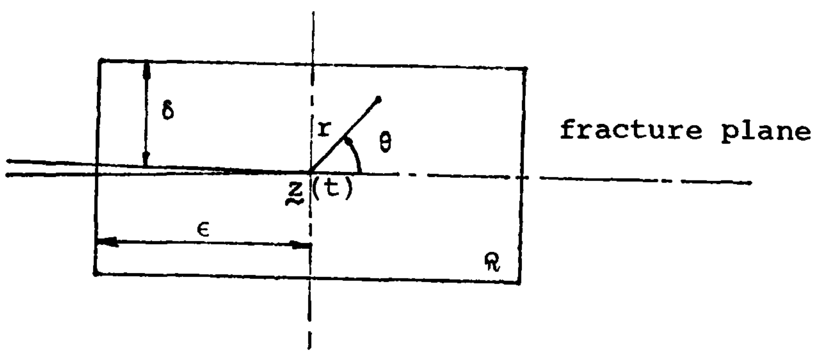

Figure 1

[1]The first evaluation of (1.3) was given in the Appendix of Achenbach and Nuismer [6] and in the corresponding "Erratum" [7].

which was established with the aid of Parseval's formula for the two-sided Laplace transform, where H is the Heaviside unit step function.

Freund's interpretation, however, is at most formal. Even if the value of $\mathcal{E}$ is the same for all loops as they are shrunk to the crack tip, we cannot, in general, conclude that

$$\lim_{\epsilon\to 0}\lim_{\delta\to 0}\int_{\partial\mathcal{R}} \underset{\sim}{s}(\underset{\sim}{x},t)\cdot\dot{\underset{\sim}{u}}(\underset{\sim}{x},t)\,ds = -\lim_{\epsilon\to 0}\int_{C_t} \underset{\sim}{s}(\underset{\sim}{x},t)\cdot\dot{\underset{\sim}{u}}(\underset{\sim}{x},t)\,ds_{\underset{\sim}{x}}, \quad (1.4)$$

where $C_t = C_t(\epsilon)$ is the portion of the fracture plane contained in $\mathcal{R}$. The independency of loops guarantees that the left-hand side of (1.4) gives $\mathcal{E}(t)$ correctly,[1] but does not say anything about the validity of the right-hand side, since $\underset{\sim}{s}\cdot\dot{\underset{\sim}{u}}$ is not, in general, integrable on $\partial\mathcal{R}$ uniformly in $\delta \geq 0$. Further, the value of the right-hand side vanishes, unless you consider generalized functions, since the integrand vanishes almost everywhere on C_t.

In spite of this defect in the mathematical proof, this method is interesting and has advantages, if it is correct, not only in that it requires a knowledge of $\underset{\sim}{s}$ and $\dot{\underset{\sim}{u}}$ only on C_t, but also in that <u>it may be valid for more general materials</u>.

It is important to note that Achenbach's argument in support of (1.2) is not necessarily confined to linear elastic materials, and the interpretation of Freund discussed above, which uses the two-sided Laplace transform to evaluate the ill-defined integral,

[1] For a more precise proof, cf. Gurtin and Yatomi [8], Theorem 1 and the Remark following it.

might also be valid for more general materials, since his flux integral expression for $\mathcal{E}$ has the property. In fact, the form of the integrand in (1.2) suggests the validity of this expression independent of material considerations; on the other hand, since (1.1) involves an expression for work which is valid only for linear behavior, (1.1) is probably not generic.

It is the purpose in this paper to examine (1.1) and (1.2) for quasi-static crack extension in a non-linear elastic material.

2. Examination in a class of non-linear elastic materials.

To make the discussion simple, we consider the quasi-static, finite and anti-plane shear field analyzed by Knowles [10]. The material was assumed to belong to the special subclass of incompressible elastic materials defined by the constitutive equation

$$w = \frac{\mu}{2b} \{ (1 + \frac{b}{n} |\nabla u|^2)^n - 1 \}, \qquad (*)$$

where w is the strain energy per unit undeformed volume, μ is the infinitesimal shear modulus, n is a hardening parameter, b is a material constant, and u is the out-of-plane displacement.

In the case of anti-plane shear, to evaluate (1.2) we need the first-order asymptotic solution for the Piola stress σ_{32} and the out-of-plane displacement u in a neighborhood of the crack tip. Using Knowles' [10] solution and notation, they are

$$\sigma_{32} \sim \frac{\mu b^{n-1} A^{2n-1}}{2^{2n-1}} \frac{(2n-1)^{2(n-1)+1/2n}}{n^{4n+1/2n-7/2}} \frac{1}{r^{1-1/2n}} \quad \text{on} \quad \theta = 0 \qquad (2.1)$$

$$u \sim \frac{A}{n^{1/2-1/2n}} r^{1-1/2n} \quad \text{on} \quad \theta = \pi \qquad (2.2)$$

as $r \to 0$, where r, θ are polar coordinates at the crack tip (see Figure 1) and A is a constant related to the stress intensity factor.

Employing Parseval's formula for the two-sided Laplace transform, we obtain a generalized formula for (1.3):

$$\int_{-\infty}^{\infty} \frac{H(v)}{v^{1-p}} \frac{H(-v)}{(-v)^{1-q}} dv = \begin{cases} \dfrac{\pi}{2 \sin p\pi} , & p + q = 1 \\ 0 , & p + q > 1 \end{cases} \tag{2.3}$$

where $p > 0$ and $q > 0$.

Then, following the method of Freund [9] or Achenbach [5], that is, introducing (2.1) and (2.2) into (1.2) and using (2.3), the energy release rate $\mathcal{E}$ is given in the form

$$\mathcal{E} = \frac{A^{2n}\pi\mu|\underset{\sim}{c}|}{b^{1-n}g(n)} , \tag{2.4}$$

where

$$g(n) = \frac{(4n^4)^n \sin \pi/2n}{n^2(2n-1)^{2n-1+1/2n}}$$

and $\underset{\sim}{c}$ is the velocity of the crack tip.

Similarly, introducing (2.1) and (2.2) into (1.1), but using the relation

$$\int_{\alpha}^{\beta} (v-\alpha)^{p-1}(\beta-v)^{q-1} dv = (\beta-\alpha)^{p+q-1} \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \qquad (p,q > 0),$$

where Γ is the gamma function, we find the expression (1.1), which has been proved only for a linear elastic material, gives the same value as (2.4).

On the other hand, using the well-known relation (cf. e.g. [11])

$$\mathcal{E} = \int_{\partial P} (w\underset{\sim}{c}\cdot\underset{\sim}{n} - \underset{\sim}{s}\cdot\nabla\underset{\sim}{u}\underset{\sim}{c})\,ds$$

(which is valid for non-linear elastic materials), where P is an arbitrary regular region surrounding the tip, the energy release rate is given in the form

$$\mathcal{E} = \frac{A^{2n}\pi\mu|\underset{\sim}{c}|}{b^{1-n}f(n)}\ , \qquad (2.5)^{1}$$

where

$$f(n) = \frac{(4n^4)^n}{n(2n-1)^{2n-1}(2n^2-2n+1)}\ .$$

Since $g(n) \neq f(n)$ in general, we find that the result (2.4) does not agree with the correct value of $\mathcal{E}$ given in (2.5).

It is interesting to note, however, that since

$$\begin{aligned} &f(n),\ g(n) \to 4 \quad \text{as} \quad n \to 1 \\ \text{and} \qquad & \qquad\qquad\qquad\qquad\qquad\qquad (2.6) \\ &f(n),\ g(n) \to 2 \quad \text{as} \quad n \to 1/2, \end{aligned}$$

the differences between (2.4) and (2.5) disappear for a linear elastic material $(n = 1)$ and for an elastic material which behaves like a <u>perfectly plastic</u> material in loading $(n = 1/2)$.

The result $(2.6)_1$ is not surprising, since (1.1) is known to be valid for a linear elastic material.

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[1] We use the results (5.18) and (5.19) by Knowles [10].

3. Conclusions.

The expressions of Achenbach and Irwin lead to a value for the energy release rate which is generally incorrect, at least for finite anti-plane shear of the material defined by (*) with $n \neq 1/2, 1$. For $n = 1$, these expressions give the correct result.

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